

# Chirped-pulse oscillators: an impact of the dynamic gain saturation

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## ABSTRACT

An effect of the dynamic gain saturation on chirped-pulse oscillator was investigated. It was found, that the dynamic gain saturation causes strong perturbations of the pulse front that destabilizes an oscillator. As a result, the chirped-pulse exists only within some limited range of dispersions and there is a limit of energy growth for a given resonator period.

**Keywords:** Femtosecond laser pulses, Chirped-pulse oscillator, Solid-state laser

## 1. INTRODUCTION

Oscillators directly generating femtosecond pulses with energy exceeding 100 nJ are of interest for numerous applications including frequency conversion, metrology, micro-machining, etc. Nowadays oscillators operating at MHz-repetition rates have reached the micro-Joule pulse energy frontier without an extra-resonator amplification. An ingenious idea allowing such an advance is the pulse stretching, that suppresses the instabilities inherent to the high-peak-power oscillators.<sup>1</sup> Since the pulse is soliton in the negative group-delay-dispersion (GDD) regime, its stretching is linearly irreversible. The issue of pulse compressibility can be solved by a chirped-pulse oscillator (CPO) providing: i) sufficient pulse stretching (few picoseconds), ii) broad spectrum (more than 100 nm), and iii) pulse compressibility down to few tens of femtoseconds.<sup>2</sup>

The remaining issue is that a high-energy oscillator (including CPO) is stable only within a confined region of GDD and pulse energy. The existing theory<sup>3,4</sup> predicts that some minimum positive GDD is required for the CPO stabilization against the CW-amplification and this “threshold” GDD increases with the energy. It was found, that the gain saturation is a key factor determining the oscillator stability. However, the existing models do not take into account the gain dynamics and, thereby, some substantial features of the regime remain unexplored. For instance, it is unclear why the region of GDD and energy, where a single stable pulse exists, is confined.<sup>5</sup>

Here, for the first time to our knowledge, the theory of CPO taking into account the gain dynamics will be presented. As it will be shown, the gain dynamics provides an explanation for the confinement of the pulse stability region. The model demonstrates an existence of two main mechanisms of pulse destabilization: the satellite rise in front of the pulse and the CW-amplification.

## 2. CPO MODELING

The CPO conception is based on the pulse energy ( $E$ ) scaling with the scaling of resonator period ( $T_{cav}$ ):  $E = P_{av}T_{cav}$  ( $P_{av}$  is the averaged power). Since the pulse peak power ( $P(0)$ ) scales with the energy:  $P(0) \approx E/T$  ( $T$  is the pulse width), one has to stretch the pulse (i.e., to increase  $T$ ) in order to reduce  $P(0)$  and, thereby, suppress the instabilities causing by the nonlinear effects inside a resonator (e.g., the self-focusing inside an active medium). However, in the solitonic regime (i.e., when the GDD is negative), such a stretching is linearly irreversible. This means that the femtosecond high-energy pulse can be produced by only nonlinear compression, which worsens its characteristics (e.g., enhances the noise). In the CPO, the picosecond pulse has an extremely wide spectrum due to large chirp. As a result, the pulse is linearly compressible down to  $\approx 1/\Delta$  ( $\Delta$  is the pulse spectrum width). The experiments demonstrate that such a regime is very stable and reproducible.<sup>2</sup>

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The pulse stability in the CPO can be explained by the combined action of two mechanisms: a pure phase mechanism and a frequency-dissipative one. The first mechanism is the balance of phase contributions from the pulse envelope  $A(t)$ :  $\beta \partial^2 A(t)/\partial t^2$  ( $\beta$  is the GDD coefficient,  $t$  is the local time ranging from  $-T_{cav}/2$  to  $T_{cav}/2$ ) and the time-dependent phase  $\phi(t)$ :  $-\beta A(t) [\partial \phi(t)/\partial t]^2$ .<sup>6</sup> Such a balance is possible if the pulse is chirped. However, a pure phase balance is not sufficient for pulse stabilization as the pulse spreads, i.e. some dissipative effects are required to form a quasi-soliton. The pulse lengthening due to GDD can be compensated by its shortening owing to frequency filtering if the pulse is chirped.<sup>7,8</sup> The chirp results in frequency deviation at the pulse front and tail, and the filter cuts off the high- and low-frequency wings of the pulse, thus the pulse shortens.

The CPO can be described on the basis of the distributed generalized complex cubic-quintic nonlinear Ginzburg-Landau model:<sup>3,4,9</sup>

$$\frac{\partial A(z,t)}{\partial z} = \left[ -\sigma(z,t)A(z,t) + (\alpha - \beta) \frac{\partial^2 A(z,t)}{\partial t^2} \right] + [(\kappa - i\gamma)P(z,t) - \kappa\zeta P(z,t)^2] A(z,t). \quad (1)$$

Here  $z$  is the propagation distance normalized to the cavity length (i.e., the cavity round-trip number, in fact).  $P(z,t) = |A(z,t)|^2$  is the instant power,  $\alpha$  is the square of the inverse spectral filter bandwidth. The parameters  $\gamma$  and  $\kappa$  are the self-phase and self-amplitude modulation coefficients, respectively. Parameter  $\zeta$  describes saturation of the self-amplitude modulation. In a broadband solid-state oscillator, for example, parameter  $\alpha$  is the square of the inverse gain band width multiplied by the saturated gain coefficient. Parameters  $\kappa$  and  $\zeta$  are defined by the Kerr-lens mode locking mechanism.<sup>3,4</sup>

The  $\sigma$ -parameter is the spectrally independent saturable net-loss coefficient, which depends on the instant pulse energy  $E(z,t) \equiv \int_{-T_{cav}/2}^t P(z,t') dt'$ . Below, such a dependence will be referred as the dynamic gain saturation.

It was found,<sup>4</sup> that a heavily-chirped pulse in CPO can be described as the soliton-like solution of Eq. (1) in the limit of time-independent  $\sigma$ . In this case, the simplest energy-dependence of the  $\sigma$ -parameter is

$$\sigma(E) \approx \delta \left( \frac{E}{E^*} - 1 \right), \quad (2)$$

where  $\delta \equiv E^* \frac{d\sigma}{dE} \big|_{E=E^*}$ ,  $E \equiv \int_{-T_{cav}/2}^{T_{cav}/2} P(t') dt'$  and  $E^* \equiv P_{av} T_{cav}$ . The spectrum of the chirped soliton-like pulse has a Lorentz profile truncated at some frequency  $\pm\Delta/2$ , where the central frequency  $\omega = 0$  corresponds to the CPO carrier frequency.<sup>4</sup> The chirped soliton-like pulse is stable within the region of its existence, i.e. within the region of  $\sigma > 0$ .<sup>4</sup> Positivity of  $\sigma$ , i.e. stability against the CW-amplification (vacuum stability), can be provided by a certain minimum positive GDD growing with energy.

Nevertheless, the existing theory does not take into account the time dependence of  $\sigma$  provided by the dynamic gain saturation. Let us consider some numerical estimations. The gain saturation fluency for a Ti:sapphire oscillator is  $E_s \equiv h\nu/\sigma_g \approx 0.8$  J/cm<sup>2</sup> ( $\sigma_g$  is the emission cross-section,  $\nu$  is the emission frequency,  $h$  is the Planck's constant). Then, the gain variation due to gain saturation per one resonator round-trip is  $T/SE_s\gamma \approx 0.2$  ( $T=1$  ps, the mode area  $S$  equals to  $130 \mu\text{m}^2$ ,  $E=200$  nJ; and the peak power is close to the stability limit, which is defined by the self-phase modulation coefficient  $\gamma = 4.5$  MW<sup>-1</sup>). That is the gain variation per one cavity round-trip is not negligible unlike that in a low-energy femtosecond oscillator operating in the solitonic regime.

Below, the extended theory of CPO taking into account the dynamic gain saturation will be considered.

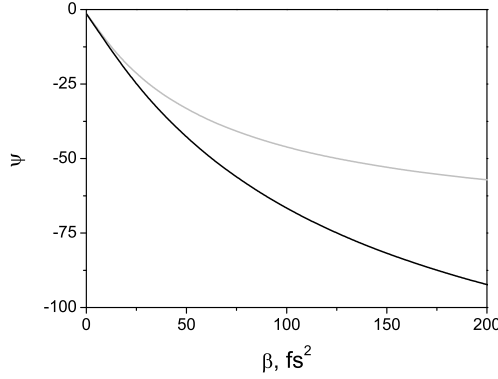


Figure 1. Dependence of the chirp  $\psi$  on GDD for the model (1,3,4) with  $\zeta = 0$  and  $\alpha = 2.5 \text{ fs}^2$ .  $\kappa = 0.02\gamma$  (black) and  $0.04\gamma$  (gray).

### 3. ANALYTICAL THEORY OF CPO WITH THE DYNAMIC GAIN SATURATION

The cubic nonlinear limit of Eq. (1) admits the chirped soliton-like solution:<sup>7, 10</sup>

$$A(t) = \sqrt{P(0)} \text{sech}[(t - vz)/T]^{1+i\psi} \exp[i\omega(t - vz) + i\phi z], \quad (3)$$

which is frequency ( $\omega$ )- and phase ( $\phi$ )-shifted and time-delayed ( $v$ ) in the presence of the dynamic gain saturation ( $\psi$  is the dimensionless chirp):

$$\sigma(z, t) = l - g(vz) + \frac{1}{E_s} \int_{vz}^t P(z, t') dt'. \quad (4)$$

The pulse parameters are:

$$\begin{aligned} \psi &= \frac{3(\kappa/\gamma - \beta/\alpha) - \sqrt{9(\beta^2/\alpha^2 + \kappa^2/\gamma^2) + 8(1 + (\kappa\beta/\gamma\alpha)^2) - 2\kappa\beta/\gamma\alpha}}{2(1 + \beta\kappa/\gamma\alpha)}, \\ v &= (3\alpha\psi/E_s + 2\alpha\omega\psi\gamma - \psi^2\beta/E_s + 2\beta/E_s + 2\omega\beta\gamma)/\gamma, \\ \omega &= \frac{\psi(\beta\psi^2 - 3\alpha\psi - 2\beta)}{2\gamma\alpha E_s(1 + \psi^2)}, \quad T^2 = \frac{\alpha(1 - \psi^2) - 2\beta\psi}{\alpha\omega^2 + \sigma_0}, \quad P(0) = \frac{\beta(\psi^2 - 2) - 3\alpha\psi}{\gamma T^2}. \end{aligned} \quad (5)$$

In Eq. (4)  $l$  is the net-loss coefficient,  $g(vz)$  is the gain at the pulse peak taking into account its time-delay with propagation,  $\sigma_0 = l - g(vz)$ . One can see a clear manifestation of the dynamic gain saturation: the time-delay and the frequency shift appear.

Fig. 1 shows the GDD-dependence of the chirp parameter  $\psi$  for two fixed  $\kappa$ . One can see, that the chirp ( $|\psi|$ ) increases with GDD and its value is sufficiently large to provide an efficient compression (the pulse width after compression is  $\approx T/|\psi|$ ). Growth of the self-amplitude modulation decreases  $|\psi|$  due to reduction of the relative contribution of the self-phase modulation in comparison with the self-amplitude modulation.

Fig. 2 (unshaded area) shows the region, where the chirped pulse exists for the model (1,3,4) with  $\zeta = 0$ . One can see, that there exists some minimum (threshold) positive GDD providing the chirped pulse generation (like the case without the dynamic gain saturation). The threshold GDD decreases with the self-amplitude modulation growth (the  $\kappa$ -growth). Also, one can see that the simplest analytical model does not predict the pulse destabilization with the GDD growth (i.e., the stability region confinement), as it takes a place in the experiment.<sup>5</sup>

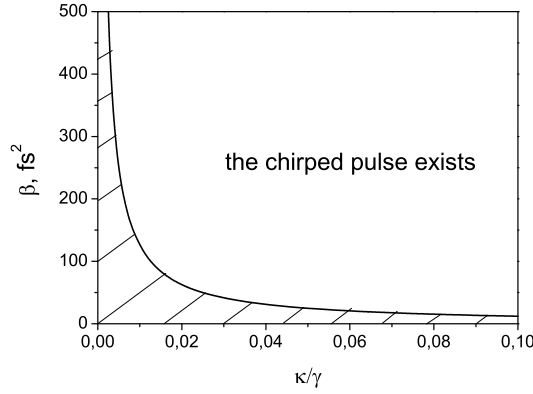


Figure 2. Region of the chirped pulse existence for the model (1,3,4) with  $\zeta = 0$  and  $\alpha = 2.5 \text{ fs}^2$ .

#### 4. NUMERICAL THEORY OF CPO WITH THE DYNAMIC GAIN SATURATION

It is clear, that the dynamic gain saturation in the form of Eq. (4) as well as the assumption of  $\zeta = 0$  do not allow taking into account the substantial features of CPO dynamics. Hence, the numerical simulations based on Eq. (1) and

$$\frac{\partial g(t)}{\partial t} = \frac{P_p}{S_p} \frac{\sigma_a}{h\nu_a} (g_{\max} - g(t)) - \frac{P(t)}{SE_s} g(t) - \frac{g(t)}{T_r} \quad (6)$$

are required. The last equation takes into account the gain dynamics in a quasi-two-level (four-level, in fact) active medium.<sup>11</sup>  $P_p$  is the absorbed pump power,  $S_p$  is the pump beam area inside an active medium,  $\sigma_a$  is the absorption cross section of active medium,  $\nu_a$  is the pump frequency,  $g_{\max}$  is the maximum gain,  $T_r$  is the gain relaxation time.

The numerical challenge is that  $\sqrt{\alpha} \ll T_{cav}$  and the integro-differential system (1,6) has to be solved on the grid containing  $2^{21}$  points for  $T_{cav} \approx 20 \text{ ns}$ . The propagation step  $\Delta z$  in the simulation equals to  $1/50$  of the cavity round-trip length.

The main result of numerical analysis is shown in Fig. 3. Figure presents the stability region on the “absorbed pump power – GDD” plane. One can see, that the stability region is confined: there are some minimum and maximum GDDs as well as minimum and maximum pump powers providing the stable chirped pulse generation. Both minimum and maximum stabilizing GDDs depend on the pump non-monotonically. That is there exists some optimal pump power ( $\approx 5.5 \text{ W}$  in our case) providing the pulse stability within the broadest range of GDDs. When the pump exceeds some maximum value ( $\approx 10 \text{ W}$  in our case), the pulse cannot be stabilized by only dispersion adjustment. It should be noted, that the existence of both maximum GDD and pump, which cannot be exceeded without the pulse destabilization, is not explicable in the framework of the previous models. An appearance of these stability limits can be interpreted as a result of the gain dynamics in a high-energy CPO.

Fig. 4 illustrates the power profiles of both stable ( $A$  and  $B$  corresponding to the points in Fig. 3) and unstable ( $C$  and  $D$ ) pulses. Within the limits of the stability region, the GDD growth broadens the pulse (transition from  $A$  to  $B$  in Fig. 4, see, also, solid curve in Fig. 5), and reduces the peak power. The pulse energy does not change substantially, but the spectral width decreases (dashed curve in Fig. 5, see, also  $A$  and  $B$  in Fig. 6).

The spectra have a profile, which is typical for the CPO: the parabolic top with the truncated edges ( $A$  and  $B$  in Fig. 6). In contrast to a situation with no dynamic gain saturation, the spectra is frequency shifted. This shift results in a partial compensation of the time-advance caused by the dynamic gain saturation. Indeed,

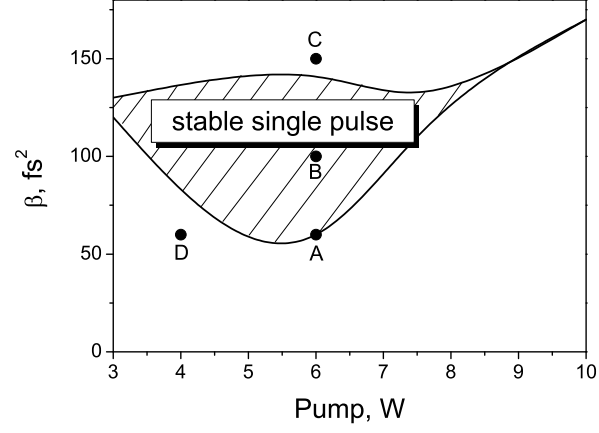


Figure 3. Region of the chirped pulse stability (shaded area) for the numerical model (1,6) with  $\zeta = 0.6\gamma$ ,  $\kappa = 0.04\gamma$ ,  $g_{max} = 0.33$ ,  $l = 0.22$ ,  $S_p = 100 \mu\text{m}^2$ ,  $S = 130 \mu\text{m}^2$ ,  $T_r = 3.5 \mu\text{s}$ . Double transit through the Ti:sapphire crystal per one round-trip ( $T_{cav} = 21 \text{ ns}$ ) was considered.

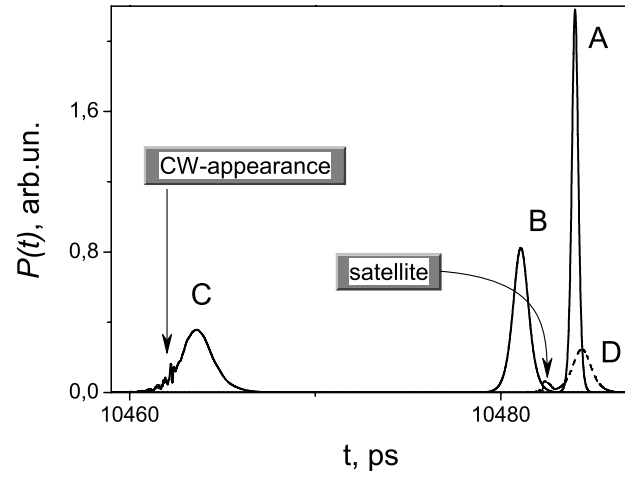


Figure 4.  $P(t)$ -profiles corresponding to the parameters specified by the points A, B, C and D in Fig. 3.

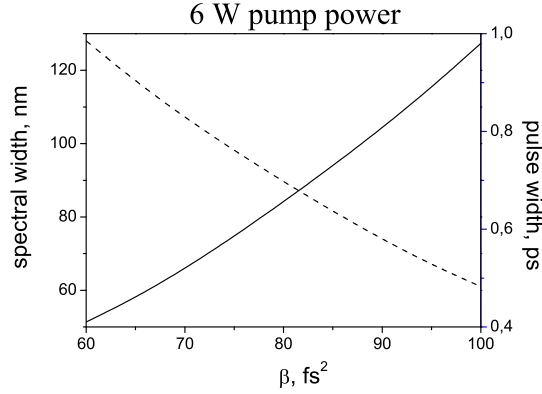


Figure 5. Dependencies of the pulse width (solid curve) and the spectrum width (dashed curve) on GDD for  $P_p = 6$  W.

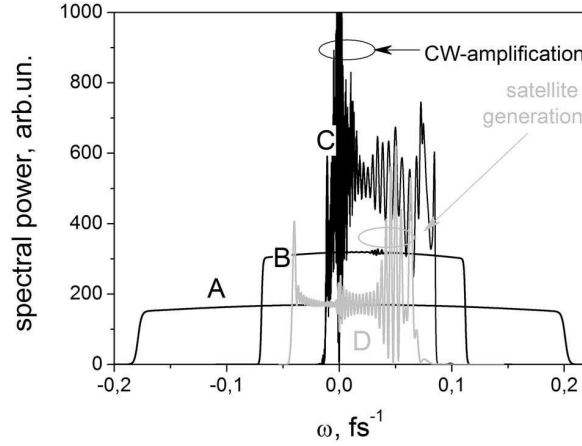


Figure 6. Spectral profiles corresponding to the parameters specified by the points  $A$ ,  $B$ ,  $C$  and  $D$  in Fig. 3.

the last leads to the primary amplification of the pulse front. The compensation of this effect in the positive dispersion domain is possible if the pulse spectrum is blue-shifted.

When the GDD crosses the upper stability border, the ripples at the pulse front appear ( $C$  in Fig. 4). Such ripples grow and destroy the pulse. The spectrum ( $C$  in Fig. 6) demonstrates, that such a destabilization corresponds to the CW-amplification in front of the pulse.

On the other hand, both GDD and pump decrease destabilizes the pulse, as well ( $D$  in Figs. 3,4,6). The destabilizing mechanism is an appearance of satellite in front of the pulse (dashed curve in Figs. 3). As a result of the dynamic gain saturation, the energy transfer from the pulse to satellite destabilizes the CPO.

## 5. CONCLUSION

To determine the operational range of CPO in terms of dispersion and energy variations, an effect of the dynamic gain saturation on the CPO stability has been studied numerically for the first time to our knowledge. It has been found, that the dynamic gain saturation has a strong impact on a CPO dynamics in contrast to a low-energy (10 nJ) mode-locked solid-state oscillator. First, there is some minimum positive net-GDD providing the oscillator stabilization. Below this value the pulse is destroyed by excitation of the satellite before the pulse. Such a satellite is clearly visible in the experiment and can co-exist stably with the pulse within some narrow

GDD-range. Second, there is some maximum GDD and its excess fragments the pulse front owing to the CW-amplification. Third, the highest pulse energy at a given repetition rate is limited because the maximum and minimum GDDs stabilizing the pulse merge.

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